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Tunneling Radiation of the Charged Particles for Charged Spherical Black Hole in VGM

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Applying Parikh's semi-classical tunneling method, Hawking radiation of charged massive particles via tunneling from charged spherical black hole in vacuum for Vector Graviton Metric theory (VGM) of gravitation is investigated. Because the derivation respects conservation of energy and charge, the tunneling rate of particles is relevant to the change of Bekenstein-Hawking entropy and the exact spectrum is not precisely thermal. The result employs an underlying unitary theory.

KEY WORDS: black hole; Quantum radiation; conservation of energy and charge.
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1. INTRODUCTION

Quantum radiation of black hole plays a tremendously important role in understanding quantum phenomenon of particle creation in curved spacetime. Hawking did pioneering works in black hole radiation in 1970s. He proved that black hole has thermal radiation and radiation spectrum is precisely thermal (Hawking, 1974). After that several methods of Hawking radiation are derived by other authors and the thermodynamic properties of static black hole, stationary black hole and non-stationary black hole have been extensively investigated (Damour, 1976; Zhao *et al.*, 1991; Zhu *et al.*, 1994; Yang *et al.*, 1996–2002; Hawking, 2005). Those works play great positive effects on the conception and further research on the black hole. But most derivations of Hawking radiation in the literatures relied on quantum field theory on a fixed space-time background. So thermal radiation spectrums they obtained are all purely thermal. When the black hole is totally evaporated, information will be lose, this means the pure quantum state change into the state of chaos. From the view of quantum field theory, ingoing state is pure, outgoing state is chaos, and unitary principle in quantum mechanics

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has been violated. Taken literally, Hawking's result implies the loss of unitary or, to put it more dramatically, the breakdown of quantum mechanics. Until Kraus (1997, 2000) and Parikh (2000) *et al.* put forward a semi-classical method to investigated Hawking radiation as tunneling in the recent year, variational space-time background is considered. In 2000, Parikh calculated tunneling radiation of uncharged particle for Schwarzschild and Reissner-Nordström black hole (Parikh *et al.*, 2000). The radiation spectrum he obtained is not a strictly thermal and can reduce to Hawking spectrum when the quadratic term is neglected, which indicates the quantum tunneling rate of black hole is a good correction to Hawking's pure thermal spectrum. Following Parikh's method, tunneling radiations of some other black holes (such as Kerr and Kerr-Newman Black hole) have been investigated (Parikh, 2004; Zhang *et al.*, 2005; Yang *et al.*, 2005). But usually, only uncharged massless particle is discussed. Obviously, it's necessary to research the tunneling rate of charged massive particles. In this paper, we extend the Parikh's work to investigate the Hawking radiation of the charged massive particles via tunneling from charged spherical black hole in vacuum for Vector Graviton Metric theory of gravitation (hereafter VGM), which is a new metric derived by Qian *et al.* (2005) in recent year. In general, a replacement of G will yield the corresponding result in VGM for the metric in vacuum. Hence, it is interesting to investigate the radiation of this black hole by using the new method. During the analysis of tunneling radiation in this paper, the self-gravity effect and conversation of energy and charge will be taken into account.

2. TUNNELING PROCESS OF THE CHARGED MASSIVE PARTICLE

The metric for the charged spherical black hole in VGM in natural unit with the metric sign $(-, +, +, +)$ has the form (Qian *et al.*, 2005)

$$ds^2 = -\left[\left(1 - \frac{m}{r}\right)^2 + \frac{Q^2}{r^2}\right] dt_V^2 + \left[\left(1 - \frac{m}{r}\right)^2 + \frac{Q^2}{r^2}\right]^{-1} \times dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{1}$$

Where and are the mass and charge of the black hole. t_V is the time coordinate of charged spherical black hole in VGM. The related electromagnetic potential is given by, $A_\mu = (A_t, 0, 0, 0)$, where

$$A_t = -\frac{Q}{r} \tag{2}$$

There is a coordinate singularity at event horizon in line element (1), so it is necessary to eliminate the singularity and choose a coordinate which is satisfied with tunneling condition. (The coordinate we want must be flat Euclidean space in radial to constant-time slices). Usually, authors introduce Painlevé coordinates transition $dt_V = dt \pm f(r)dr$ (Painlevé *et al.*, 1921), where

$$f(r) = \frac{r\sqrt{2mr - m^2 - Q^2}}{(r-m)^2 + Q^2}.$$

So line element (1) becomes,

$$ds^2 = -\left[\left(1 - \frac{m}{r}\right)^2 + \frac{Q^2}{r^2}\right]dt^2 \pm 2\left[1 - \left(1 - \frac{m}{r}\right)^2 - \frac{Q^2}{r^2}\right]^{1/2} \times dtdr + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \tag{3}$$

It is manifest that there is no singularity at event horizon. From Eq. (3), we can obtain the equation of the radial null geodesics satisfies as. $d\theta = d\varphi = 0, ds^2 = 0$. i.e.

$$-\left[\left(1 - \frac{m}{r}\right)^2 + \frac{Q^2}{r^2}\right]dt^2 \pm 2\left[1 - \left(1 - \frac{m}{r}\right)^2 - \frac{Q^2}{r^2}\right]^{1/2} dtdr + dr^2 = 0 \tag{4}$$

So, the radial null geodesics is given by

$$\dot{r} = 1 \pm \sqrt{1 - \left(1 - \frac{m}{r}\right)^2 - \frac{Q^2}{r^2}} \tag{5}$$

Equation (5) is the radial null geodesics for massless particles because trajectory of particle is like-light in the form of S-wave. When tunneling particle is massive, the relationship of group velocity and group velocity must be considered. According to de Broglie's hypothesis, from the definition of the phase velocity v_p and the group velocity v_g (Jiang *et al.*, 2006), we have

$$v_p = \frac{1}{2} v_g \tag{6}$$

Since the tunneling process is an instantaneous effect, the metric in the line element (3) satisfies Landau's condition of the coordinate clock synchronization, the coordinate time difference of two events, which take place simultaneously in different places, is

$$dt = -\frac{g_{tr}}{g_{tt}} dr_c \quad (d\theta = d\varphi = 0). \tag{7}$$

Where dr_c is the location of the tunneling particle. So the group velocity can be expressed as

$$v_g = \frac{dr_c}{dt} = -\frac{g_{00}}{g_{01}} = \pm \frac{(r-m)^2 + Q^2}{2\sqrt{mr^3 - m^2 r^2 - Q^2 r^2}} \tag{8}$$

Therefore the phase velocity (the radial geodesics) is

$$\dot{r} = v_p = \frac{1}{2} v_g = \pm \frac{(r-m)^2 + Q^2}{2\sqrt{mr^3 - m^2r^2 - Q^2r^2}} \tag{9}$$

where "±" correspond to the outgoing and ingoing geodesics. According to radiation scenario (Parikh, 2004), when a virtual particle pair is created just inside the horizon, the positive energy virtual particle can tunnel out (no classical escape route exists) where it materializes as a real particle. Alternatively, for a pair created just outside the horizon, the negative energy virtual particle, which is forbidden outside, can tunnel inwards. In either case, the negative energy particle is absorbed by the black hole, resulting in a decrease in the mass of the black hole. Respect to energy conservation and charge conservation, we consider the total ADM mass and charge of space-time are fixed and the mass and charge of black hole are allowed to fluctuate. When a particle with the shell of energy ω and charge q evaporates from the event horizon, the mass and charge of black hole will become $m-\omega$ and $Q - q$, respectively. Considering the charged massive particle tunnels out of the event horizon along the radial direction, we can get the new radial geodesics of the black hole at the event horizon.

$$\dot{r} = \frac{[r-(m-\omega)]^2 + (Q-q)^2}{2\sqrt{(m-\omega)\, r^3 - (m-\omega)^2\, r^2 - (Q-q)^2\, r^2}} \tag{10}$$

The related electromagnetic potential becomes

$$A_t = -\frac{Q-q}{r} \tag{11}$$

Now, let's move on to discuss the tunneling radiation characteristics across the event horizon of the black hole. In the semi-classical limit, applying Wentzel-Kramers-Brillouin (WKB) approximation, the relation of the probability of the radiation particles as tunneling and the imaginary part of action is (Parikh, 2000; Zhang *et al.*, 2005)

$$\Gamma \sim e^{-2ImS}. \tag{12}$$

Where S is the action for trajectory. When the self-gravitation is taken into account, the imaginary part of the action for charged massive particles can be expressed as

$$\begin{aligned} \operatorname{Im} S = \operatorname{Im} &= \int_{t_i}^{t_f} \left(L - P_{At}\dot{A}_t\right) \operatorname{Im} \int_{r_i}^{r_f} \left(P_r \dot{r} - P_{At}\dot{A}_t\right) \frac{dr}{\dot{r}} \\ &= \operatorname{Im} \int_{r_i}^{r_f} \int_{(0,0)}^{(P_r, P_{At})} \left(\dot{r}\, dP_r - \dot{A}_t\, dP_{At}\right) \frac{dr}{\dot{r}} \end{aligned} \tag{13}$$

where i and f represent the parameters of event horizon before and after the particle tunnel out. According to Hamilton's canonical equation of motion, we have

$$\dot{r} = \left.\frac{dH}{dP_r}\right|_{r;A_t,P_{A_t}}, \quad dH|_{r;A_t,P_{A_t}} = d(m-\omega)$$
$$\dot{A}_t = \left.\frac{dH}{dP_{A_t}}\right|_{A_t;r,P_r}, \quad dH|_{A_t;rP_{A_t}} = \frac{Q-q}{r}d(Q-q) \tag{14}$$

Substituting Eqs. (10) and (14) into Eq. (13) and switching the order of integral, we obtain

$$\begin{aligned} \operatorname{Im} S &= \operatorname{Im}\int_{r_i}^{r_f}\int_{(m,Q)}^{(m-\omega,Q-q)} [dH|_{(r;A_t,P_{A_t})} - dH|_{(A_t;r,P_r)}]\frac{dr}{\dot{r}} \\ &= \operatorname{Im}\int_{r_i}^{r_f}\int_{(m,Q)}^{(m-\omega,Q-q)} \frac{2\sqrt{(m-\omega)r^3-(m-\omega)^2r^2-(Q-q)^2r^2}}{[r-(m-\omega)]^2+(Q-q)^2} \\ &\quad\times\left[d(m-\omega)-\frac{Q-q}{r}d(Q-q)\right]dr \end{aligned} \tag{15}$$

Since $[r-(m-\omega)]^2/r^2-(Q-q)^2/r^2=0$ satisfies the horizon equation after the particle with energy ω and charge q tunnels out, there is a single pole in Eq. (15). Let us carry out integral by deforming the contour around the pole so as to ensure that positive energy solutions decay in time, and get

$$\operatorname{Im} S = \operatorname{Im}\int_{r_i}^{r_f}(-i\pi r)dr = -\frac{\pi}{2}\left(r_f^2-r_i^2\right). \tag{16}$$

Substituting Eq. (16) into Eq. (12), we get the relationship between the tunneling rate and imaginary part of particle action is

$$\Gamma \sim e^{-2\operatorname{Im}S} = e^{\pi\left(r_f^2-r_i^2\right)} = e^{\Delta S_{BH}} \tag{17}$$

where S_{BH} is the Bekenstein-Hawking entropy at the event horizon, and

$$\Delta S_{BH} = S_{BH}(m-w) - S_{BH}(m) \tag{18}$$

From Eq. (17) One can find that the tunneling probability is related to the change of Bekenstein-Hawking entropy. This result is same to Parikh's tunneling picture, which might provide a mechanism to deal with the information loss paradox.

Similarly, when we calculate radiation spectrum of the ingoing particles with energy ω and charge q across the cosmic horizon, the parameter $m-\omega$ should be replaced by $m+\omega$ in the Eqs. (10) and (14), and the charge parameter $Q-q$ will be replaced by $Q + q$ in the Eqs. (11) and (14). The tunneling rate we obtain is the same to get Eq. (17), which indicate that there is a same conclusion to event horizon. Surely, if we substitute Eqs. (5) and (14) into Eq. (13) the tunneling rate of massless particle can be gotten (Yang *et al.*, 2005).

3. SUMMARY

We have investigated the tunneling radiation of the charged massive particle from charged spherical black hole in VGM. Due to the conservation of the energy and charge are considered, black hole radiation causes the space-time background geometry varies, variations of event horizon and cosmic horizon of black hole are associated. The radiation spectrum is not strictly thermal. As the tunneling rate in quantum mechanics is obtained by

$$\Gamma (i \to f) \sim \left|A_{if}\right|^2 \alpha. \tag{19}$$

where $|A_{if}|^2$ is the square of the amplitude for the tunneling action, $\alpha = N_f/N_i$ is the phase space factor, N_i, N_f are the number of the initial and final microcosmic states, respectively. $S_i \sim \mathrm{In}\, N_i \to N_i \sim \mathrm{e}^{S_i}$, $\mathrm{In}\, N_f \to N_f \sim \mathrm{e}^{S_f}$, hence

$$\Gamma \sim \frac{e^{S_f}}{e^{S_i}} = e^{S_f - S_i} = e^{\Delta S}. \tag{20}$$

Obviously, Eq. (20) is consistent with the result our obtained by applying Parikh's semi-classical quantum tunneling method. So Eq. (17) satisfies the underlying unitary theory in quantum mechanics, which gives a good explanation to information loss of black hole radiation. Comparing with references (Yang *et al.*, 2005; Zhang *et al.*, 2005) we find that the conclusion of thermal radiation is the same for charged or uncharged particle in static spacetime, which is a great support to Parikh's theory. Since the charged spherical black hole in VGM have thermal radiation and radiation spectrum is consistent with that of black holes which are not in VGM well, which indicate the radiation of other black holes in VGM have the similar result. In summary it is possible to carry information during the radiation of the black hole and Parikh's semi-classical method provides a rational explanation to the puzzle of black hole information loss.

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